

Active Control on the Torsional Vibration of an Auxiliary Power Unit during Engine Cranking Using the Gaussian Pseudo- spectral Method

Hou Zhi-chao¹, Yan Ming-gang², Zhong Bi-qing³

Tsinghua University, State Key Laboratory of Automotive Safety and Energy, Beijing 100084

¹ houzcz@tsinghua.edu.cn, ² ymg_chd@163.com, ³ 707272882@qq.com

Summary

Torsional vibration analysis and active control are carried out on the auxiliary power unit (APU) of a series-parallel hybrid electric city transit bus during engine cranking. A lumped parameter model was established for the torsional vibration of the APU system, and nonlinear equations were then derived in the state space by introducing analytically expressed engine excitations. In order to reduce vibration and energy consumption, an optimal control problem was constructed, to which a Gaussian pseudo-spectral method was adopted for solutions. Results and discussions clearly show the control effect on the torsional vibration of the system.

Key words: hybrid electric vehicle, powertrain, controller, vehicle performance

1 Introduction

For a traditional vehicle driven by an internal combustion engine (ICE), torsional vibration of the driveline is a big concern with respect to vehicle comfort and structure safety. For a hybrid vehicle, there is at least one motor incorporated in the driveline, and there are usually two motors with the most common configurations. One motor is an integrated starter and generator (ISG), which can act as a motor to crank the engine or as a generator to charge the battery. Another one is the driving motor (TM) used to drive the vehicle when necessary. A torsional damper is usually put between the engine and the ISG, and the three components together form a subsystem usually called as the accessory power unit (APU). Dramatic torsional vibration might occur to an APU, as the system will run through its resonant area when the ISG cranks. Due to the motors, the torsional vibration of a hybrid powertrain is much more complex than its traditional counterparts^[1]. On the other hand, the motors create possibility to actively control the torsional vibration of the whole power train or its subsystem.

Hybrid vehicle has been a hot topic for quite a few years, but published researches about active vibration control of the driveline in a hybrid vehicle are not too much. Among the available researches, some tried to reduce torsional vibration of the APU by adjusting the output torque of the integrated starter/generator (ISG) or a belt-driving starter and generator (BSG), while some others used the main driving motor to suppress the torsional vibration of the afterwards components. A brief review will be presented in the follows. It should be noted that only a few papers are summarized here due to limited space.

In 2007 Ito et al achieved vibration control on the Toyota hybrid powertrain THS-II using two controllers^[2]. One controller suppresses the engine speed ripple by changing the output torque of the generator according to engine torque fluctuation that was figured out based on calculated crankshaft angles. With another controller, calculation was firstly conducted for the difference between the estimated wheel speed and the motor speed, and a proportional control was adopted to compensate motor torque to finally reduce the

torsional vibration of the shaft after the motor. It should be noted in the study that the torque variation due to reciprocating inertia force of the engine is neglected in calculating the engine speed fluctuation. The shaft angle of the torsional damper was not taken into consideration either. In 2009 Syed also presented researches on driveline control using two controllers^[3]. With one controller, the difference is calculated between the target speed and actual speed of the generator, and a PI control strategy is employed to adjust the output torque of the generator so as to control the engine speed. Using the second controller, the inertia torque of the generator is calculated, and then used as feedforward to change the output torque from the generator to reduce the torque fluctuation of the wheel. According to the difference between the actual speed and target speed of the engine Njeh et al^[4] tried to suppress the rotational speed fluctuation of the engine by compensating the generator torque using a PI control strategy. In 2013, Kou et al^[5] presented a slide mode control to cut down the torsional vibration of a APU system, where the compensating torque was determined according to the target and actual speeds of the motor.

The torsional vibration of the APU contributes a lot to the vibration a hybrid driveline. In the system, both the engine exciting torque and the damper stiffness are nonlinear. As a result, those PID based control is not an optimal control, although they are easier to implement and can reduce the vibration to some extent. According to the above observation, this paper tries to address analysis and active control on the torsional vibration of the APU in a series-parallel hybrid electric city transit bus during engine cranking. An optimal control problem is defined for the system, and solved numerically by adopting a Gaussian pseudo-spectral method.

2 Torsional Vibration Model of the APU

2.1 Equation of motion

The APU to be investigated consists of an engine (ICE), a damper and a motor (ISG), as shown in Figure 1. Notating the torques applied at the engine and the motor by T_{ice} and T_{isg} , respectively.

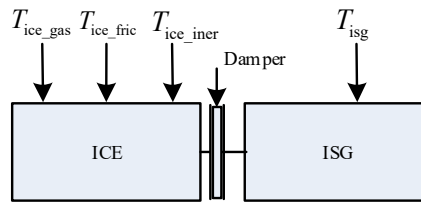


Figure1: Schematic of the APU

The torque acted on the engine is divided into the torque from inertia of the moving components T_{ice_iner} , the torque from the gas T_{ice_gas} , and the torque from friction T_{ice_fric} , and one has $T_{ice} = T_{ice_gas} + T_{ice_iner} + T_{ice_fric}$.

For such a system, the governing equation for the torsional vibration of the system can be readily derived as

$$\begin{cases} T_{ice} - (\varphi_1 - \varphi_2)k_1(\varphi_1, \varphi_2) - c(\dot{\varphi}_1 - \dot{\varphi}_2) = J_1\ddot{\varphi}_1 \\ T_{isg} + (\varphi_1 - \varphi_2)k_1(\varphi_1, \varphi_2) + c(\dot{\varphi}_1 - \dot{\varphi}_2) = J_2\ddot{\varphi}_2 \end{cases} \quad (1)$$

In the equation, φ_1 and φ_2 are the rotating angles of the engine and ISG motor while J_1 and J_2 are the corresponding inertia moments of rotating parts. c and $k_1(\varphi_1, \varphi_2)$ are the damping efficient and stiffness rate of the torsional damper. The stiffness is usually a piecewise linear function of relative angle $\varphi_1 - \varphi_2$, as will be shown later.

2.2 Engine torque and its approximation

2.2.1 Engine torque at general situation

The engine torque will be determined if its aforementioned three components are respectively figured out. The gas torque can be expressed as

$$T_{ice_gas} = -\frac{\pi D^2 (p_{gas} - p_0)}{4 \cos \phi} r \sin(\varphi_1 + \phi), \quad \phi = \arcsin\left(\frac{r}{l} \sin \varphi_1\right) \quad (2, 3)$$

Here p_{gas} and p_0 are the gas pressure and the standard atmospheric pressure, respectively, r is the crank radius, D is the cylinder diameter while ϕ and l are the angle and length of the connecting rod.

The torque from the inertia of moving components is usually calculated by

$$T_{ice_iner} = -M_{rec} \frac{r \sin(\varphi_1 + \phi)}{\cos \phi} \ddot{x} \quad (4)$$

with M_{rec} as the equivalent piston mass and $\ddot{x}$ the two order differentiation of the piston position with respect to time. Assuming

$$x = r \cos \varphi_1 + l \cos \phi = r \cos \varphi_1 + l \sqrt{1 - \left(\frac{r}{l} \sin \varphi_1\right)^2} \quad (5)$$

it can be figured out that

$$M_{rec} = M_{piston} + M_{rod} \frac{l_1}{l} \quad (6)$$

Here M_{rec} and M_{piston} are respectively the mass of the piston and the mass of the connecting rod, while l_1 is the distance from the centroid of the connecting rod to the central axis of the crank.

There are usually two classes of models to describe the engine friction torque T_{ice_fric} . The first class can be called as average friction model, or steady model. By such class of models, the friction torque varies only with the rotating speed, throttle opening and the lubricating oil temperature. If these parameters are kept constant, the friction torque will be constant when the crank angle varies within 720°. The second classes are transient models which can describe friction variation with respect to the crank angle within 720°. The most popular ones are the Rezeki-Henein model, the simplified Rezeki-Henein model and models put forward by Taraza and et al^[6].

2.2.2 Engine torque during motor cranking

For the hybrid vehicle, the engine is cranked by the ISG motor to achieve a cold start. The procedure will be called motor cranking later in this paper. During the procedure before the snifting valve works, the rotating speed is pretty low, and there is no combustion with the fuel and air mixture. With such an observation and for the sake of more effective numerical calculation, we introduce following assumptions to simplify the expression of the engine torque.

- (1) Neglect on/off procedure of the exhaust valves, and set the intake lag angle and the exhaust advance angle to be zero;
- (2) During the intake stroke and exhaust stroke, there is no much difference between the gas pressure within the cylinder and the atmosphere pressure due to the opening of the intake valve or the exhaust value. Therefore, we assume the pressure in the cylinders and in the crank box equal to a standard atmosphere pressure. As a result, the gas force acted on the piston is zero.
- (3) For combustion stroke and power stroke, the intake valve and the exhaust valve are closed, and the procedure of gas compression or expansion lasts only a short period of time. Either of the strokes can be approximated as a closed adiabatic process.
- (4) the average friction model is chosen to describe the engine friction T_{ice_fric} . According to the data measured from experiments, it can be noted that the friction torque does not change much for a fixed temperature.

With these assumption or approximation, an analytical formula can be derived for the gas torque during motor cranking procedure. The volume of the gas in a cylinder is

$$V = \frac{V_L}{6} \frac{\lambda}{\lambda - 1} - \frac{\pi D^2}{4} \left(r \cos \varphi_1 + l \sqrt{1 - \left(\frac{r}{l} \sin \varphi_1 \right)^2} \right) \quad (7)$$

where V_L is the engine displacement, and λ is the compression ratio. For the quasi-static adiabatic process of an ideal gas without power loss, the pressure and volume satisfy $pV^\kappa = \text{const}$ with κ an isentropic index. Therefore, one has $p_0 V_\alpha^\kappa = p_{\text{gas}} V^\kappa$ with cylinder α at any time during the motor cranking. As a result,

$$p_{\text{gas}} = \frac{p_0 V_\alpha^\kappa}{V^\kappa} \quad (8)$$

Substituting equation (8) into equation (2) yields an explicit expression for the gas torque. Using equation (4) for the inertia torque and the average friction model for the friction torque, the total engine torque can be calculated readily.

3 Engine Cranking and Optimal Control

In order to suppress the torsional vibration of the APU during the stage when the ISG drives the engine, a set of equation is defined in the state space as

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} x_3 \\ x_4 \\ -\frac{k_1(x_1, x_2)}{J_1} - \frac{c_1}{J_1}(x_3 - x_4) + \frac{T_{\text{ice}}(x_1, x_3)}{J_1} \\ \frac{k_1(x_1, x_2)}{J_2} + \frac{c_1}{J_2}(x_3 - x_4) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ u \end{pmatrix}, \quad \begin{cases} x_1 = \varphi_1 \\ x_2 = \varphi_2 \\ x_3 = \dot{\varphi}_1 \\ x_4 = \dot{\varphi}_2 \end{cases} \quad (9)$$

where u is the control torque to be supplied by the ISG motor. For the optimal control, a cost function is defined to be

$$J = a_1 t_f + \int_0^{t_f} \left(a_2 (x_3 - x_4)^2 + a_3 u x_4 \right) dt \quad (10)$$

In the equation, t_f is the end time of concern, $u x_4$ reflects energy consumption, and a_i ($i=1,2,3$) are weighting factors. The index covers the response time, vibration level, and energy consumption. With engine cranking by the ISG motor, the final speeds of the engine and the motor are usually predefined. It is thus clear that equations (9) and (10) define a single-stage nonlinear optimal control problem within a finite time span. In the problem, the initial time, the initial status and part of the end status are known but the end time is not known in advance. Furthermore, there are equation constraints on the state variables, and inequality constraints on the control torque.

A numerical approach, the Gaussian pseudo-spectral method is adopted to solve this optimal control problem, which is implemented by means of a tool package GPOPS of the Matlab^[7]. The key steps of the method are listed briefly as follows. According to the cost function and corresponding performance indices, a Bolza problem is firstly defined as

$$J = J_a + J_b, \quad J_a = \Phi(x(-1), t_0, x(1), t_f), \quad J_b = \frac{t_f - t_0}{2} \int_{-1}^1 g(x(\tau), u(\tau), t_0, t_f) d\tau \quad (11)$$

Here t_0 and t_f are the initial time and the end time, respectively, and τ satisfies

$$t = \frac{t_f - t_0}{2} \tau + \frac{t_f + t_0}{2} \quad (12)$$

Equation (9) is then rewritten as

$$\frac{dx}{d\tau} = \frac{t_f - t_0}{2} f(x(\tau), u(\tau), \tau; t_0, t_f) \quad (13)$$

Secondly transform the differential equation about constraints into algebraic equation by means of interpolation at Gaussian points using the Lagrange interpolation polynomial. As a result, the Bolza problem is changed to a constrained nonlinear programming problem. The related equations are listed below.

$$\sum_{i=0}^N D_{ki} x_i - \frac{t_f - t_0}{2} f(x_k, u_k, \tau_k; t_0, t_f) = 0 \quad (k=1, \dots, N) \quad (14)$$

with J and J_a the same in formula as defined in equation (11), but

$$J_b = \frac{t_f - t_0}{2} \sum_{k=1}^N w_k g(x, u, \tau_k; t_0, t_f) d\tau \quad (15)$$

In the equation, w_k is the Gaussian weighting factor.

4 Results and Discussions

The APU of a hybrid city bus is chosen to demonstrate the applicability of the aforementioned vibration control strategy. After giving the basic data of the system, we focus on the effects of various weighting factor in the cost function and the effects of the control by simulation.

4.1 Target APU and basic data

The target system is shown in Figure 1 with main parameters of the system presented in Table 1.

Table 1: Main parameters of the target APU

J_1 /kg·m ²	J_2 /kg·m ²	c /N·m/(rev/min)	D /mm	r /mm	l /mm	ε	M_{rec} /kg
1.52	2.14	0.5	105	125	260	11	1.4

Stiffness of the torsional damper is depicted in Figure 2, which is piece-wise linear function of the relative angle of the corresponding shafts.

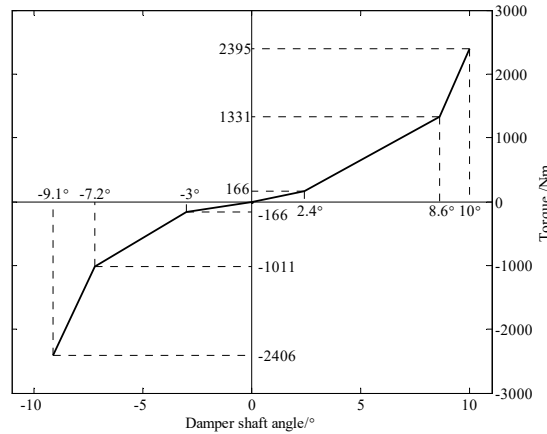


Figure 2 Stiffness function of the damper

Referring to the operation of the hybrid city bus, we can figure out the initial status, some of the final status and the feasible region of the control torque as

$$\begin{cases} x_{10} = 0 \\ x_{20} = 0 \\ x_{30} = 0 \\ x_{40} = 0 \end{cases}, \begin{cases} x_{3t_f} = 650 \\ x_{4t_f} = 650 \end{cases}, u \in [-600, 600] \quad (16)$$

In the equation, the second value or symbol in the subscript represents time of concern. In other words, the initial time is 0 when the motor starts to run, and t_f is the end time when the engine speed arrives at the predefined value. It should be noted that the units for the rotating speed and the control torque are rev/min and Nm, respectively.

4.2 Results and discussions

Assign different values to the weighting factors and pursue solutions for the optimal control problem, one can clearly know the impact of the factors to system performances. As an tentative work focusing on vibration control, we assume $a_2=1.0$ and set a_3 a very small value. Let's first check the influence of parameter a_1 . In the following cases, which implies neglecting energy consumption. Figure 3 shows the impact of parameter a_1 to the ISG torque variation and to the torsional angle of the damper, and Figure 4 shows the influence on control effect.

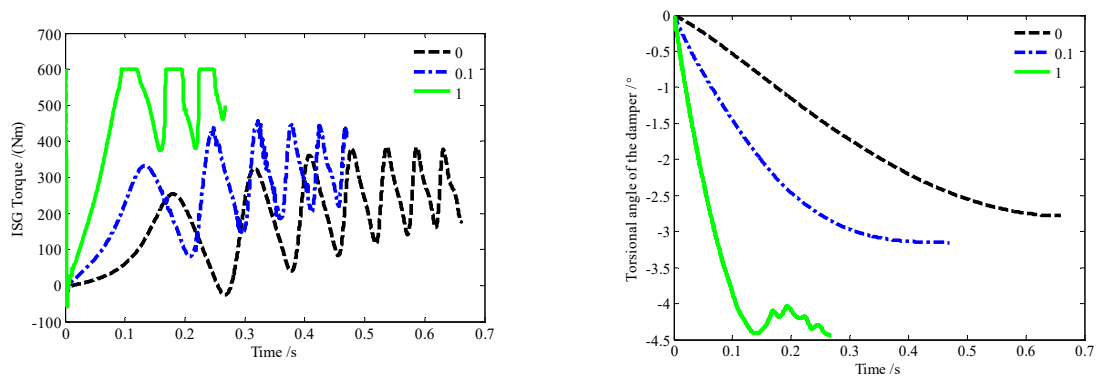


Figure 3: ISG torque (left) and torsional angle of the damper (right) vs. parameter a_1

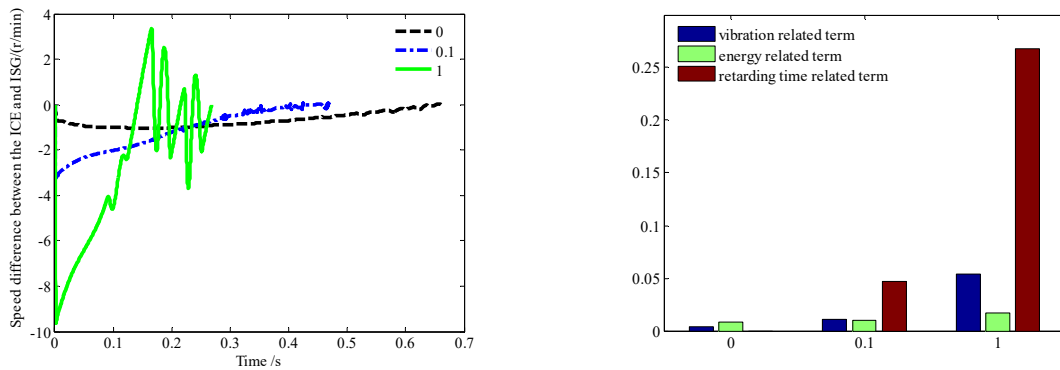


Figure 4: Variation of control performance vs. parameter a_1

From Figures 3 and 4, it can be noted that it is helpful to reduce the speed difference between the ISG motor and the engine by including the starting time as a term in the performance index when the weighting factor is small enough. For bigger weighting factor, the torque from ISG motor gets bigger and the cranking becomes faster, but the torsional angle of the damper also takes a higher value. When the weighting factor is bigger enough, say $a_1=1.0$, the ISG torque is saturated, and the speed difference between the engine and the ISG motor fluctuates. Overall, increasing a_1 means quick cranking but higher vibration and more energy consumption.

Figures 5 to 7 show the dynamic behaviour of the APU with or without active control. For the active control, $a_1=0.1$, $a_2=1.0$ and $a_3=1.0 \times 10^{-8}$.

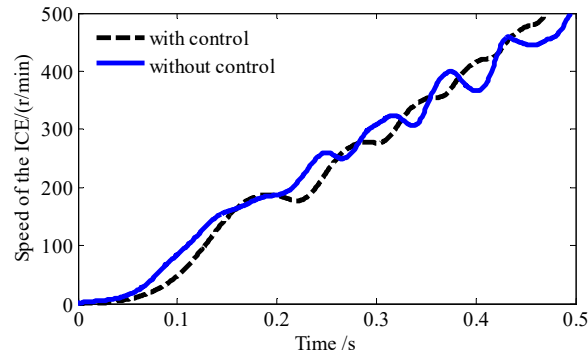


Figure 5: Variation of the engine speed: with or without control

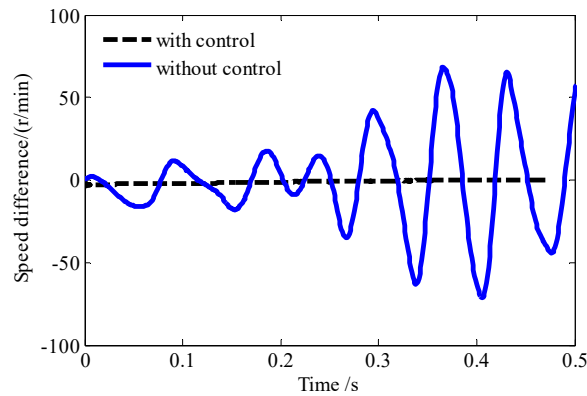


Figure 6: Speed difference between the engine and the motor: with or without control

It can be noted from the figure that the system suffers from dramatic torsional vibration due to resonance when no control is applied. Under such a situation, there is big speed difference between the engine and the ISG motor, and the torsional angle of the damper is big. With the active torque compensation on the other hand, the speeds of the engine and the motor rise smoothly and approach each other quickly. Furthermore, the control rapidly reduces the torsional angle of the damper to zero, and sharply suppress the speed fluctuation of the engine. The results also indicate that the Gaussian pseudo-spectral method can figure out the right torque compensation to reduce the torsional vibration of the target APU.

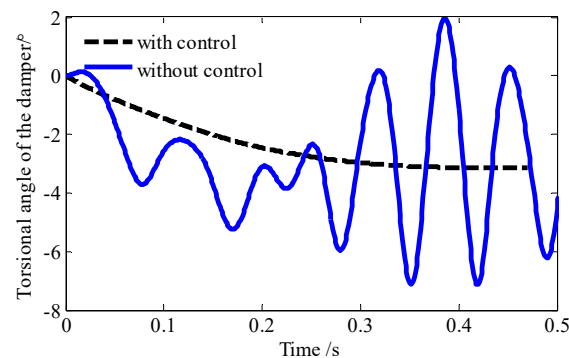


Figure 7: Torsional angle of the damper: with or without control

Summary

Equations are derived for the torsional vibration of the accessory power unit of a hybrid city bus, with nonlinearity from both the damper stiffness and the engine excitation. After making some simplification with the engine torque according to engine behaviour during cranking, an optimal control problem is then established in state space. The Gaussian pseudo-spectral method is adopted for the optimal solution of the

problem, so as to actively suppress the torsional vibration of the system. Simulations on an example system clearly show that the control strategy can effectively reduce the torsional vibration of the APU.

Acknowledgments

The research is financially supported by Research Foundation of the State Key Laboratory of Automotive Safety and Energy (grant number: ZZ2014-082).

References

- [1]. M. G Yan, Y. Zhang, Z. C. Hou, et al, *Torsional vibration response and parameter influence analysis for the entire driveline of a hybrid electric bus*, Engineering Mechanics (in Chinese), 31(2014), 223-227
- [2]. Y. Ito, S. Tomura, S. Sasaki, *Development of vibration reduction motor control for hybrid vehicles*, The 33rd Annual Conference of the IEEE Industrial Electronics Society, Taipei: IEEE, 2007, 516-521
- [3]. F.U. Syed, M.L. Kuang, Y. Hao, *Active damping wheel-torque control system to reduce drive line oscillations in a power-split hybrid electric vehicle*, IEEE Transactions on Vehicular Technology, 58(2009), 4769-4785
- [4]. M. Njeh, S. Cauet, P. Coirault, P. Martin, *Torque harmonic reduction in hybrid vehicles*, American Control Conference, Baltimore: IEEE, 2010, 5838-5843
- [5]. Y. S. Kou, F. Weslati, *Development of a hybrid powertrain active damping control system via sliding mode control scheme*, SAE International, 2013-01-0486, 2013
- [6]. S. L. Marek, N. A. Henein, *Transient engine and piston friction during starting*, SAE Technical Paper 92197, 1992
- [7]. A.V. Rao, D. Benson, G.T. Huntington G T, et al, *User's Manual for GPOPS Version 3.0: A MATLAB Software for Solving Multiple-Phase Optimal Control Problems Using Pseudospectral Methods*, 2010.

Authors



Zhi-chao Hou is a Professor in Department of Automotive Engineering at Tsinghua University. He received a B.Sc. and a M.Sc. in Mechanical Engineering from Harbin Shipbuilding Engineering Institute in 1988 and 1991, respectively. In 1995 he received a Ph.D. degree in Applied Mechanics from Tsinghua University. His current research interests include structural dynamics and vibration control, advance suspension and energy harvesting, and lightweight design.